Math 143 Sample Final Exam Problems

Question 1 For each of the following sequences $\{a_n\}$, decide whether it converges or diverges. If the sequence converges, compute the limit.

a) $a_n = \dfrac{3-4n^2+\cos n}{\sqrt[3]{5n^6-4n^5+101}}$ **Converges** **Diverges** **Limit=** ______________

b) $a_n = \sqrt{n^2+5n}-n$ **Converges** **Diverges** **Limit=** ______________

c) $a_n = \sqrt[n]{3^{2n-3}}$ **Converges** **Diverges** **Limit=** ______________

d) $a_n = \dfrac{n+(-1)^n n}{n}$ **Converges** **Diverges** **Limit=** ______________

Question 2 For each of the following series decide whether the series converges or diverges. Write the name of the test(s) used.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ **Converges** **Diverges** **Test Used=**______________

b) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin^4 n}{n^{3/2}}$ **Converges** **Diverges** **Test Used=**______________

c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}\frac{(\ln n)^{2002}}{n^{2/3}}$ **Converges** **Diverges** **Test Used=**______________

d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n^2+1}$ **Converges** **Diverges** **Test Used=**______________

Question 3 Compute the sum of the following infinite series: $\displaystyle\sum_{n=3}^{\infty} \frac{(-3)^{n-2}}{2^{3n+1}}$

Question 4 Find the interval of convergence of the power series: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (n+3)!}{n! 3^{2n}}(2x-1)^n$. Don't forget to check the endpoints!

Question 5 Use the first three non-zero terms of the power series centered at $x=0$ for $f(x) = \dfrac{\sin(2x^3)}{x^3}$ to estimate the integral $\int_0^1 f(x)\,dx$.

Question 6 Find the first three terms of the Taylor series for $f(x) = \tan x$ centered at $x = \pi/4$.

Question 7 For the parametric curve $x = (\cos t + \sin t)$, $y = (\cos t - \sin t)$, find the equation of the tangent line at the point where $t = \pi/3$. Find the length of the curve from $t = 0$ to $t = \pi/4$. Find the area of the surface of revolution gotten by rotating the curve from $t = 0$ to $t = \pi/4$ about the y–axis.

Question 8 Find the area enclosed by the cardioid $r = 2 + 2\sin\theta$. Find the equation of the tangent line to the cardioid at the point when $\theta = \pi/3$.

Question 9 Find the length of the spiral $r = \theta$ from $\theta = 0$ to $\theta = \pi/2$.

Question 10 Find the tangent line to the curve given by $\mathbf{r}(t) = (3 - 1/t^2)\mathbf{i} + \sin(\pi t)\mathbf{j} - (\ln(5-2t))\mathbf{k}$ at the point $(11/4, 0, 0)$.

Question 11 Find the equation of the plane containing the two (parallel) lines: $\mathbf{r}_1(t) = (0,1,-2) + t(1,-2,4)$ and $\mathbf{r}_2(t) = (-5,3,1) + t(1,-2,4)$.

Question 12 Find the equation of the line through the point $(3,1,-2)$ that intersects and is perpendicular to the line given parametrically as: $x = -1+t$, $y = -2+t$, $z = -1+t$.

Question 13 Let $\mathbf{u} = (a, b, 1)$, $\mathbf{v} = (1, 2, 3)$ and $\mathbf{w} = (-3, 4, 7)$. Find a value of a and b that makes $\mathbf{u}$ orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

Question 14 Find the path $\mathbf{r}(t)$ which satisfies the condition $\dfrac{d\mathbf{r}}{dt} = (t^2 - t)\mathbf{i} - (\sin t)\mathbf{j} + (16 - t^3)\mathbf{k}$ and $\mathbf{r}(0) = 3\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$.

Question 15 Find the length of the curve $\mathbf{r}(t) = (\sqrt{2}t)\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$, $0 \le t \le 2$. Find the curvature of the curve when $t = 0$.

Answers 1. a) C, $-4/\sqrt[3]{5}$ b) C, $5/2$ c) C, 9 d) D
2. a) D, LCT or IT b) C, SCT c) C, LCT d) C, AST
3. $\dfrac{-3}{(2^7)(11)}$ 4. $-4 < x < 5$ 5. $2 - \dfrac{8}{(3!)(7)} + \dfrac{32}{(5!)(13)}$ 6. $1 + 2(x - \pi/4) + 2(x - \pi/4)^2$
7. $y - \frac{1}{2}(1 - \sqrt{3}) = \frac{1+\sqrt{3}}{\sqrt{3}-1}\left(x - \left(\frac{1}{2}(1 + \sqrt{3})\right)\right)$, $L = \frac{1}{4}\sqrt{2}\pi$, $S = 2\sqrt{2}\pi$
8. $A = 6\pi$, $y - \left(\frac{3}{2} + \sqrt{3}\right) = -1\left(x - \left(1 + \sqrt{3}/2\right)\right)$
9. $L = \int_0^{\pi/2} \sqrt{1 + \theta^2}\, d\theta = \frac{1}{8}\pi\sqrt{4 + \pi^2} + \frac{1}{2}\ln(2) - \frac{1}{2}\ln(-\pi + \sqrt{4 + \pi^2})$
10. $(11/4, 0, 0) + t(\frac{1}{4}, \pi, 2)$ 11. $14x + 23y + 8z = 7$ 12. $(3, 1, -2) + t(2, 1, -3)$ 13. $a = 1/5$, $b = -8/5$
14. $\mathbf{r}(t) = (\frac{1}{3}t^3 - \frac{1}{2}t^2 + 3)\mathbf{i} + (4 + \cos t)\mathbf{j} + (16t - \frac{1}{4}t^4 - 7)\mathbf{k}$
15. $L = e^2 - e^{-2}$, $\kappa = \frac{1}{4}\sqrt{2}$ (Use Theorem 10 on page 901)